Lecture 8

Dr.
Mahmoud Khedr

Third Edition

CHAPTER

6

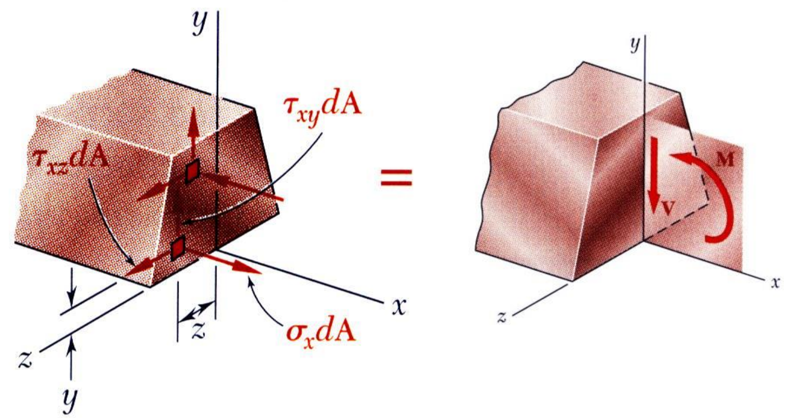
MECHANICS OF MATERIALS

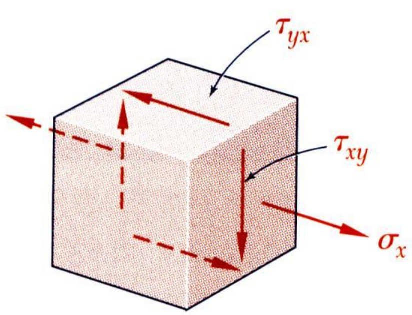
Ferdinand P. Beer
E. Russell Johnston, Jr.
John T. DeWolf

Lecture Notes:
J. Walt Oler
Texas Tech University

Shearing Stresses in Beams

Introduction





- Transverse loading applied to a beam results in normal and shearing stresses in transverse sections.

- Distribution of normal and shearing stresses satisfies

$$
\begin{aligned}
F_x &= \int \sigma_x dA = 0 & M_x &= \int \left(y\tau_{xz} - z\tau_{xy} \right) dA = 0 \\
F_y &= \int \tau_{xy} dA = -V & M_y &= \int z\sigma_x dA = 0 \\
F_z &= \int \tau_{xz} dA = 0 & M_z &= \int \left(-y\sigma_x \right) = 0
\end{aligned}
$$

- When shearing stresses are exerted on the vertical faces of an element, equal stresses must be exerted on the horizontal faces

- Longitudinal shearing stresses must exist in any member subjected to transverse loading.

Objectives

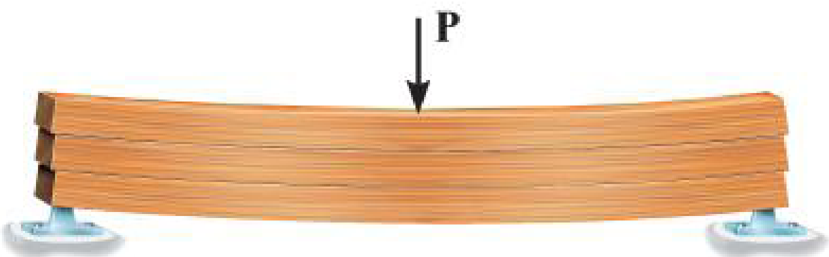
- To develop a **method for finding the shear stress in a beam** made from **homogeneous material** that behaves in a linear-elastic manner
- This method of analysis is limited to special cases of x-sectional geometry
- Discuss the concept of shear flow, with shear stress for beams and thin-walled members
- Discuss the shear center

Outline

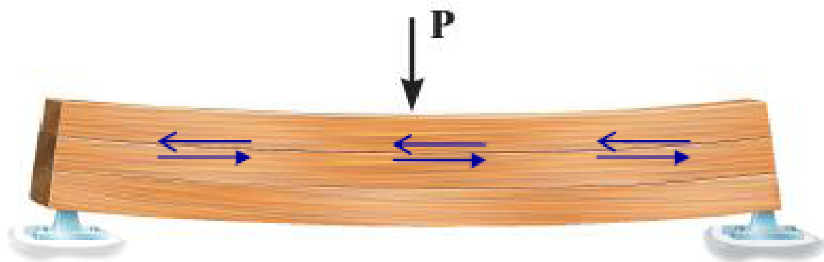
1. **To study Shear Stresses in Straight Members**
2. **To understand how to apply the Shear Formula**
3. **To determine and draw the Shear Stresses in Beams' x-sec**
4. Shear Flow in Built-up Members
5. Shear Flow in Thin-Walled Members
6. *Shear Center

SHEAR IN STRAIGHT MEMBERS

- As shown below, **if top and bottom surfaces of each board are smooth and not bonded together**, then application of load **P** will cause the **boards to slide relative to one another.**
- However, **if boards are bonded together, longitudinal shear stresses will develop** and distort x-section in a complex manner



Boards not bonded together
(a)

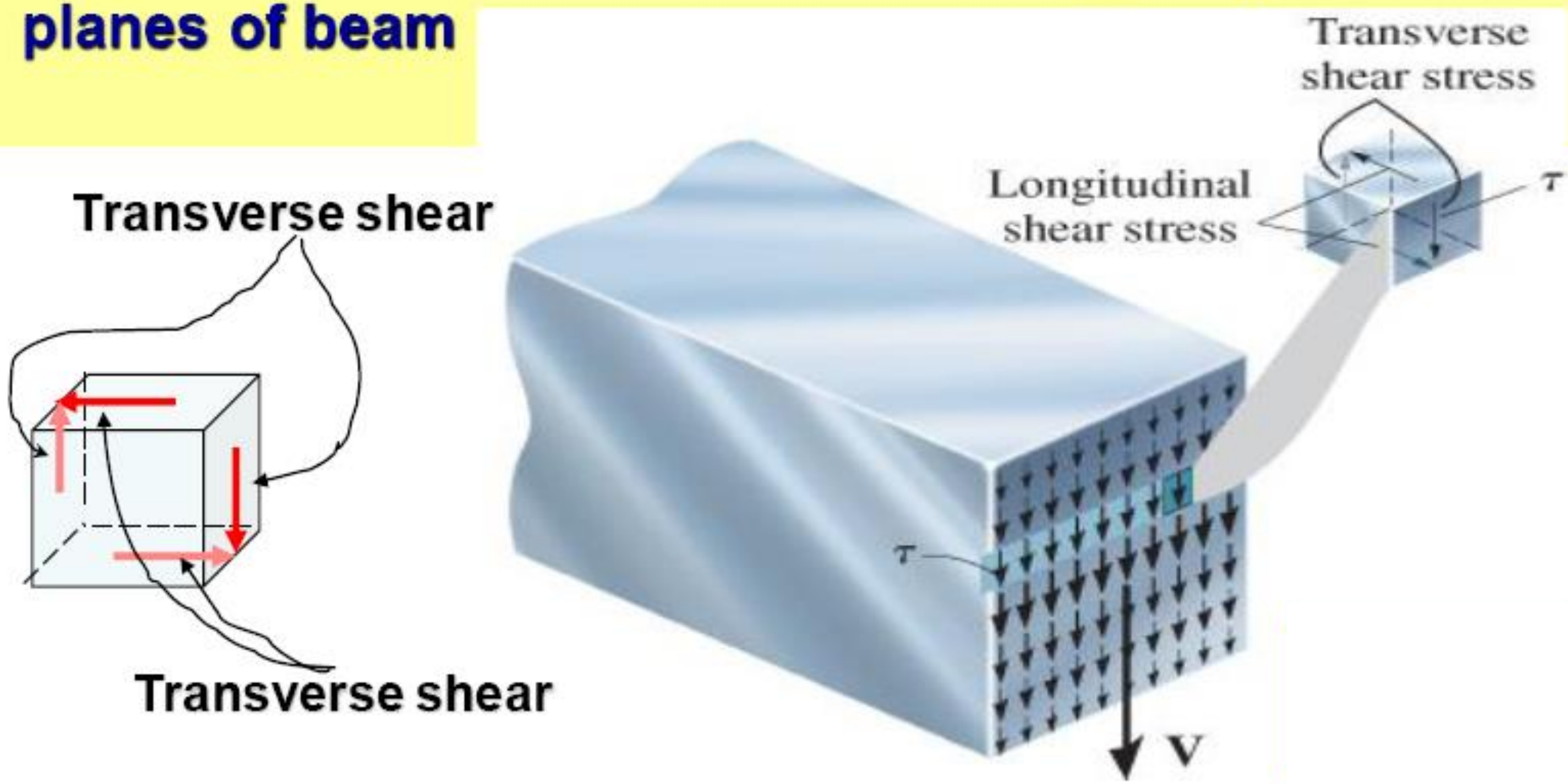


Boards bonded together
(b)

SHEAR IN STRAIGHT MEMBERS

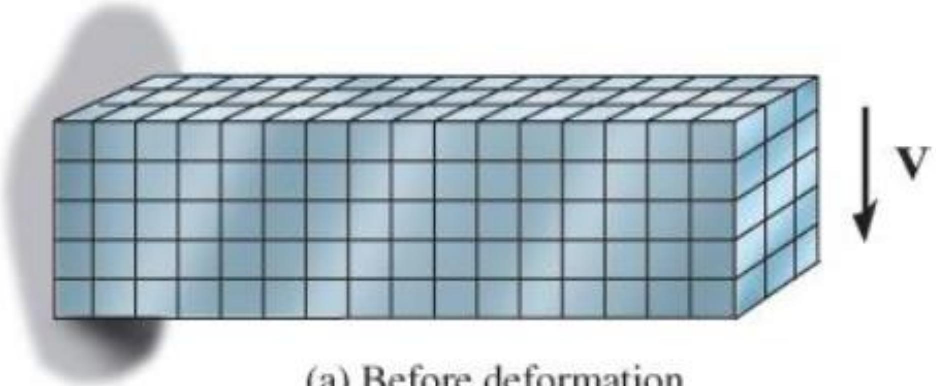
Shear **V** **is the result of a transverse shear-stress distribution that acts over the beam's x-section.**

Due to complementary property of shear, associated longitudinal shear stresses also act along **longitudinal planes of beam**

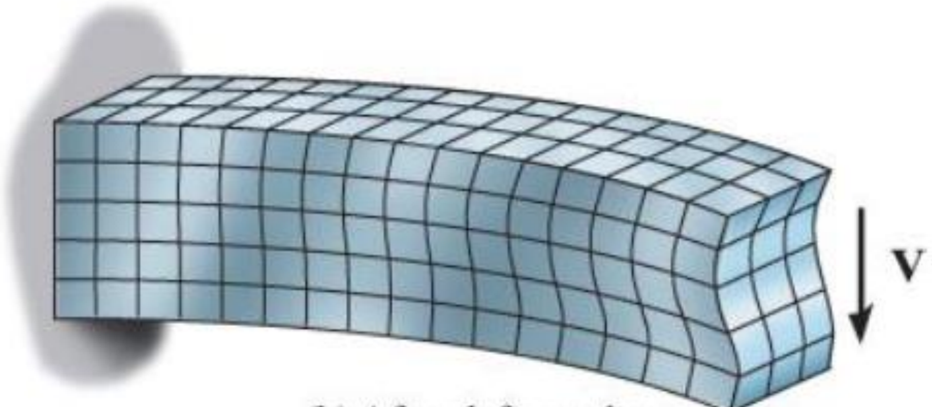


SHEAR IN STRAIGHT MEMBERS

- As shown, when shear **V** is applied, the non-uniform shear-strain distribution over x-section will cause it to *warp*, i.e., *not* remain in plane.

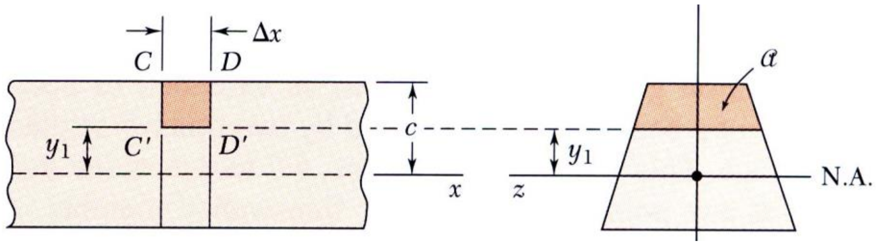


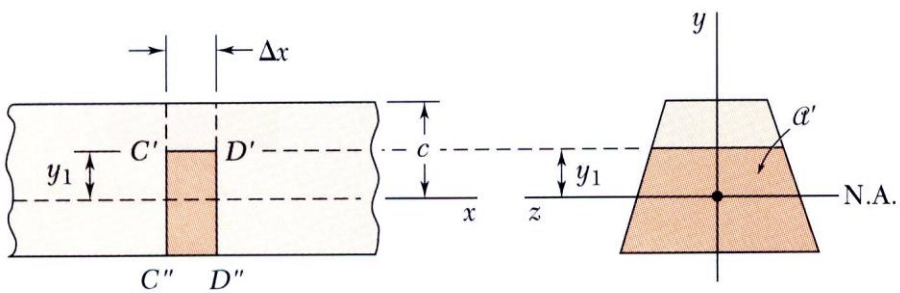
(a) Before deformation



(b) After deformation

Shear on the Horizontal Face of a Beam Element





- Shear flow,

$$q = \frac{\Delta H}{\Delta x} = \frac{VQ}{I} = shear\ \ flow$$

- where

$$Q = \int_A y\, dA$$

$$= \text{first moment of area above } y_1$$

$$I = \int_{A+A'} y^2 dA$$

$$= \text{second moment of full cross section}$$

- Same result found for lower area

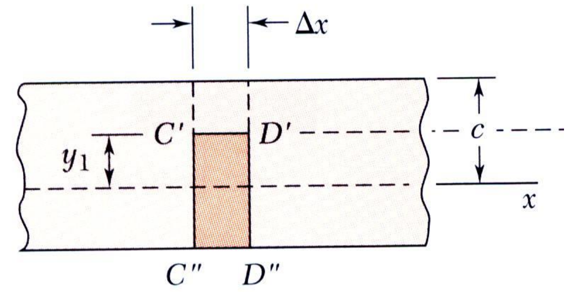
$$q' = \frac{\Delta H'}{\Delta x} = \frac{VQ'}{I} = -q'$$

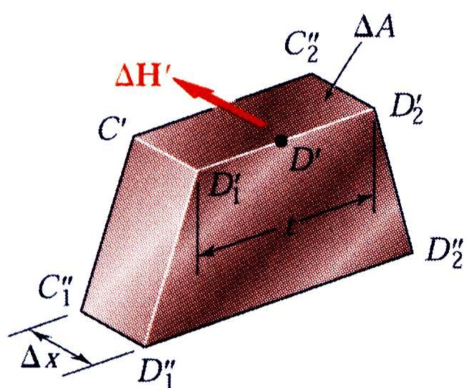
$$Q + Q' = 0$$

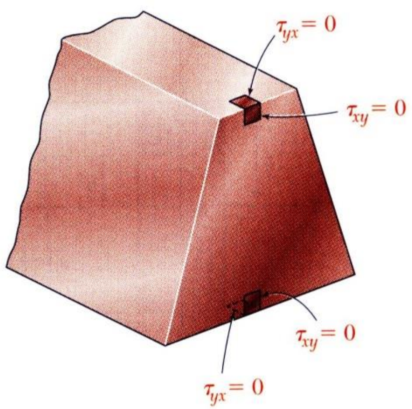
$$= \text{first moment with respect to neutral axis}$$

$$\Delta H' = -\Delta H$$

Determination of the Shearing Stress in a Beam







- The *average* shearing stress on the horizontal face of the element is obtained by dividing the shearing force on the element by the area of the face.

$$\tau_{ave} = \frac{\Delta H}{\Delta A} = \frac{q\,\Delta x}{\Delta A} = \frac{VQ}{I}\frac{\Delta x}{t\,\Delta x}$$

$$= \frac{VQ}{It}$$

- On the upper and lower surfaces of the beam, τ_{yx}= 0. It follows that τ_{xy}= 0 on the upper and lower edges of the transverse sections.

- If the width of the beam is comparable or large relative to its depth, the shearing stresses at D_1 and D_2 are significantly higher than at D.

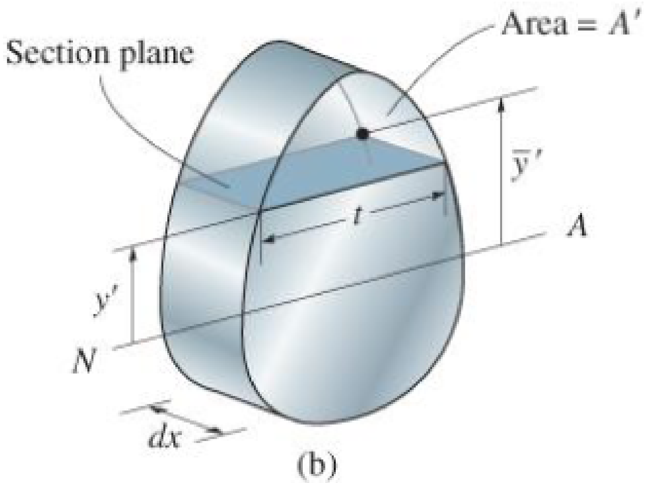
Shear Formula

- **By first principles, flexure formula and V = dM/dx, we obtain**

$$\tau = \frac{VQ}{It}$$

Equation 7-3

τ = **Shear stress in member at the pt located a distance y' from the neutral axis**. Assumed to be constant and therefore **averaged across the width t** of member



(b)

V = **Internal resultant shear force**, determined from method of sections and equations of equilibrium

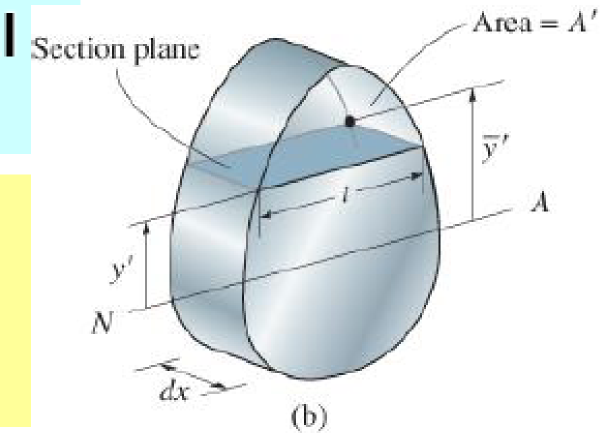
Shear Formula

- By first principles, flexure formula and $V = dM/dx$, we get:

$$\tau = \frac{VQ}{It}$$

Equation 7-3

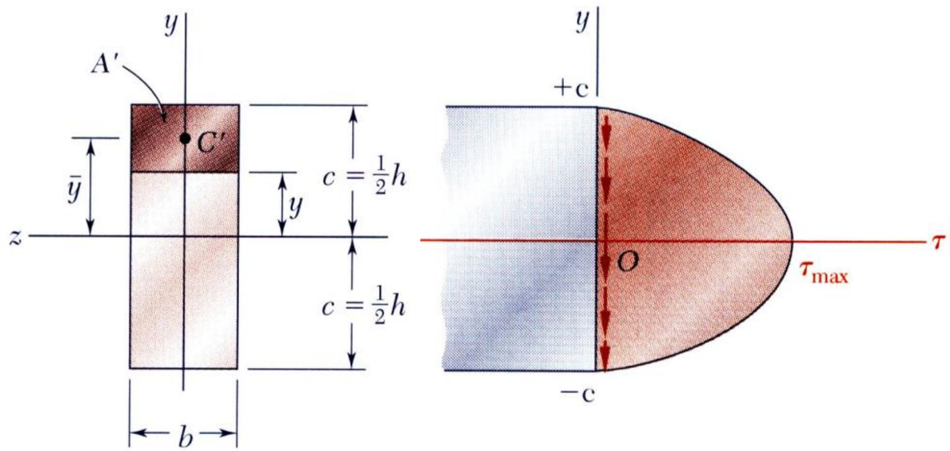
I = moment of inertia of entire x-sectional area computed about the neutral axis



t = width of the member's x-sectional area, measured at the pt where τ is to be determined

Q = 1st moment of area where A' is the top (or bottom) portion of x-sectional area, defined from section where t is measured, and y' is distance of Centroid of A', measured from neutral axis

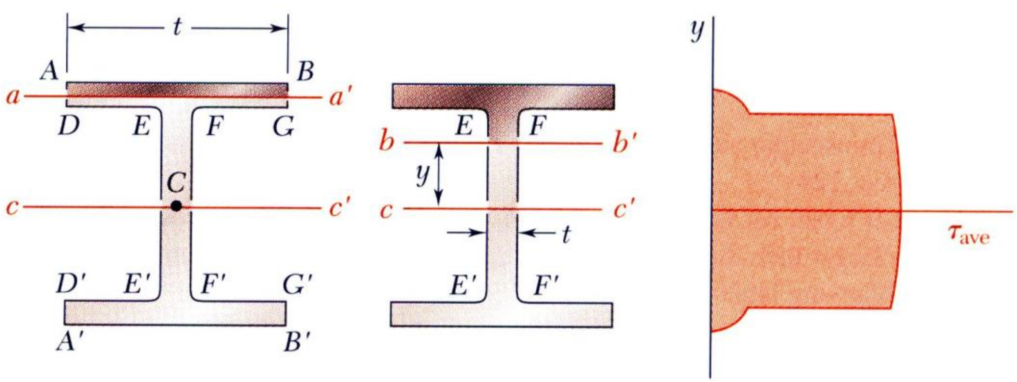
Shearing Stresses τ_{xy} in Common Types of Beams



- For a narrow rectangular beam,

$$\tau_{xy} = \frac{VQ}{Ib} = \frac{3}{2}\frac{V}{A}\left(1 - \frac{y^2}{c^2}\right)$$

$$\tau_{max} = \frac{3}{2}\frac{V}{A}$$



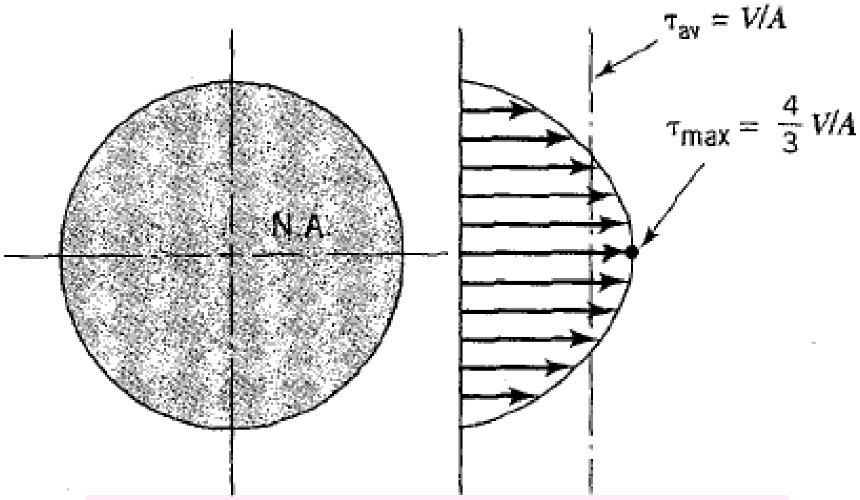
- For American Standard (S-beam) and wide-flange (W-beam) beams

$$\tau_{ave} = \frac{VQ}{It}$$

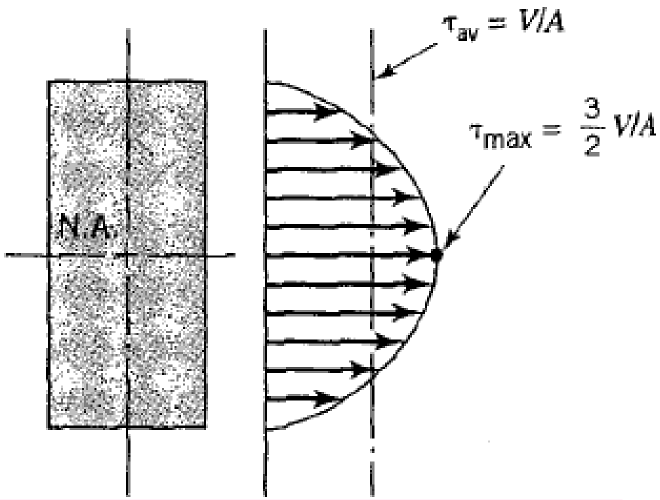
$$\tau_{max} = \frac{V}{A_{web}}$$

Shearing Stresses τ_{xy} in Common Types of Beams

For circular and rectangular cross sections, transverse shear can be estimated in terms of average shear stress



$\tau = VQ/It = 1.333$ V/A

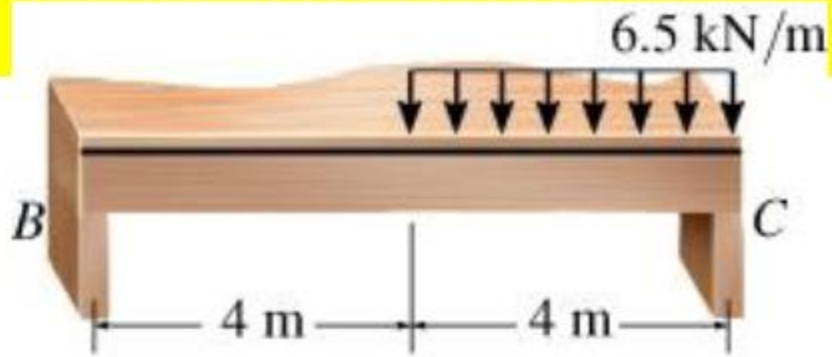


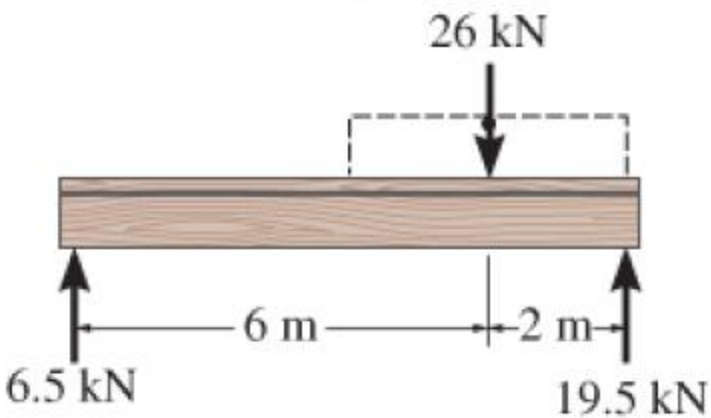
$\tau = VQ/It = 1.5$ V/A

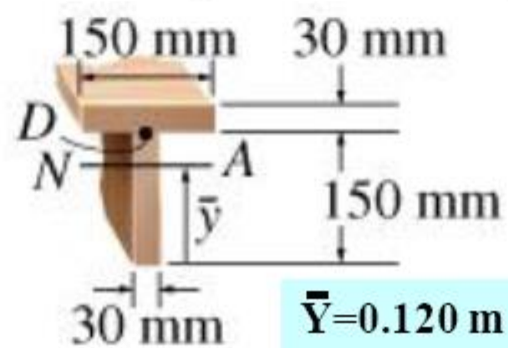
Example

Beam shown is made from two boards. **Determine the maximum shear stress** in the glue necessary to hold the boards together along the seams where they are joined. **Supports at *B* and *C* exert only vertical reactions on the beam.**

$$\tau = \frac{VQ}{It}$$







(a)

$\bar{Y} = 0.120$ m

$I = 27.0(10^{-6})$ m^4

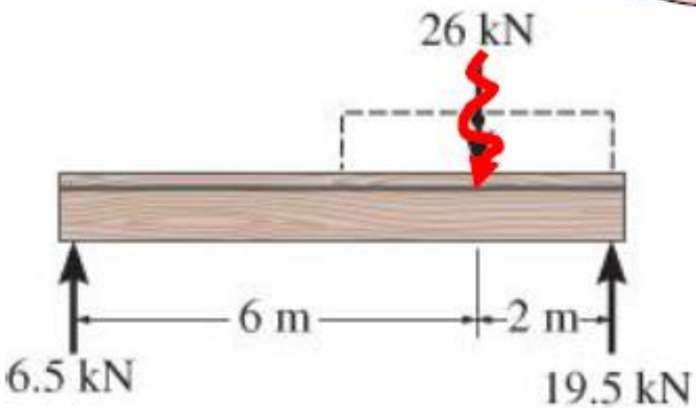
Example

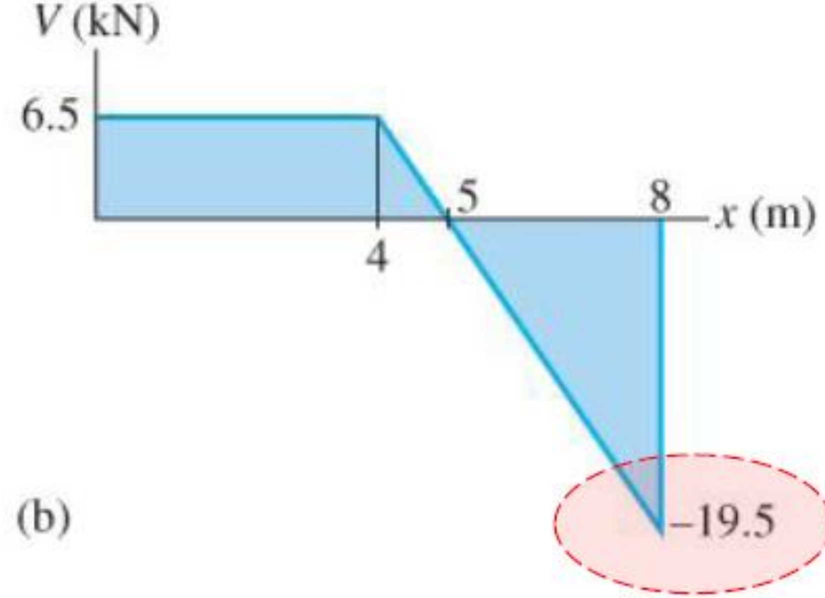
Internal shear (*V*⇒⇒*SFD*)

Find: the reactions at ***B&C***, and ***SFD*** for the beam.

Find: **Max. absolute shear in the beam is 19.5 kN.**

$$\tau = \frac{VQ}{It}$$





Example

Section properties $\bar{Y}$ & I

The centroid and therefore the neutral axis will be determined from the reference axis placed at bottom of the x-sectional area. Working in units of meters, we have

$$\tau = \frac{VQ}{It}$$

$$\bar{y} = \frac{\Sigma\, \bar{y}A}{\Sigma\, A} = \ldots = 0.120\ \mathrm{m}$$

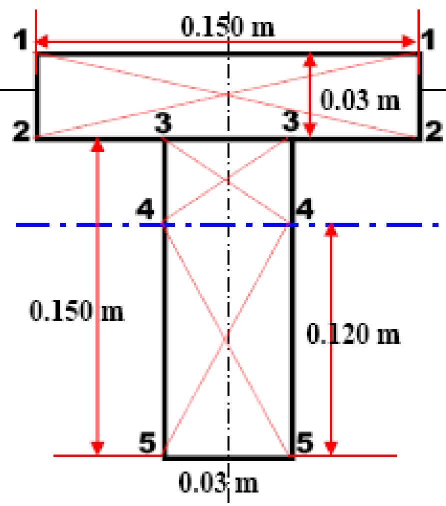
Thus, the moment of inertia, computed about the neutral axis is,

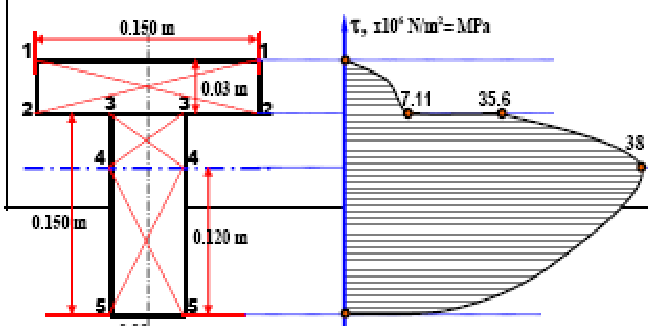
$$I = \ldots = 27.0(10^{-6})\ \mathrm{m}^4$$

Example

$V = 19.5\text{x}10^3$ N, $I = 27\text{x}10^{-6}\text{m}^4$, $V/I = 5.27\ \text{x}10^9 \text{N/m}^4$

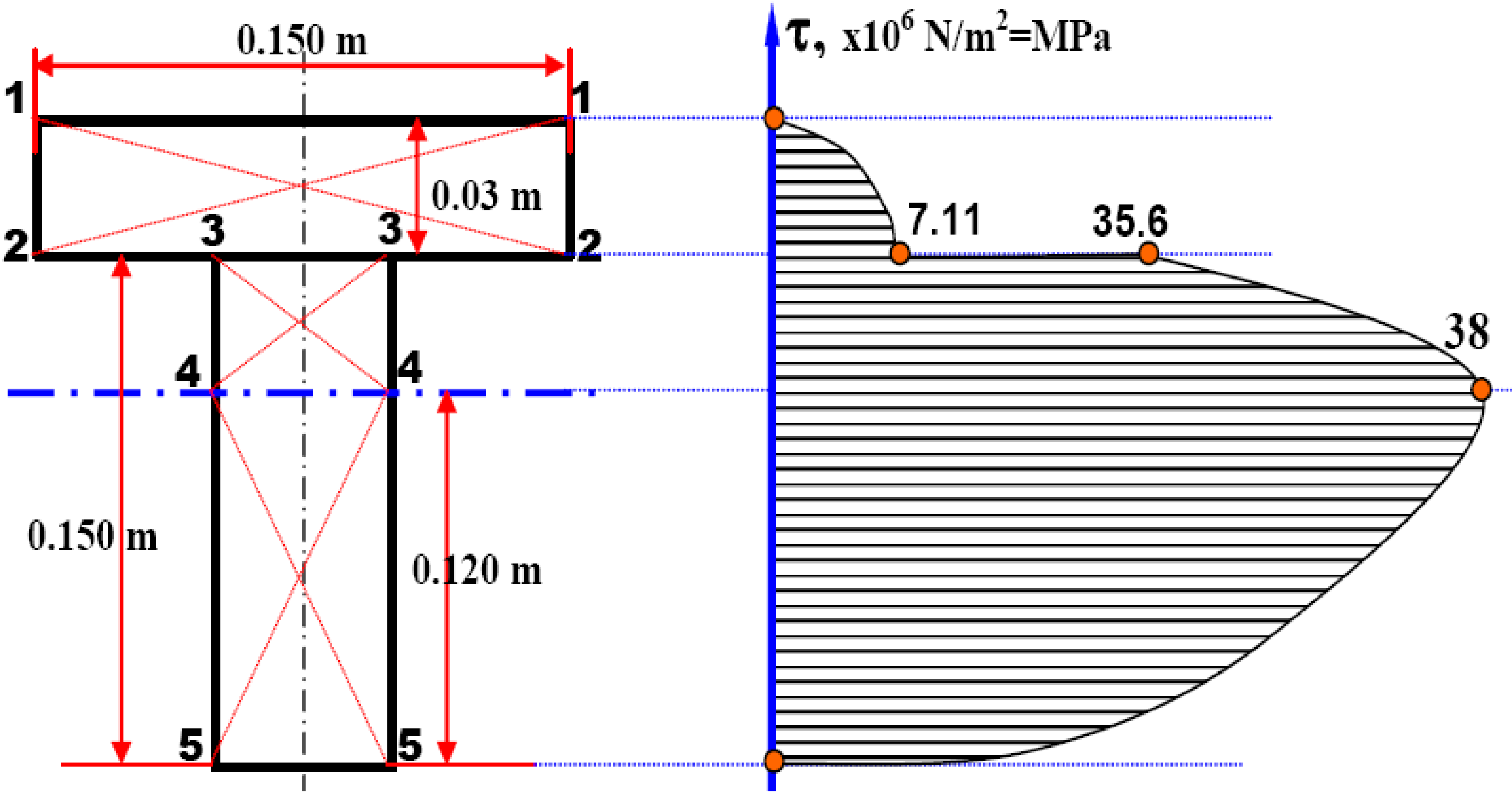
x-section



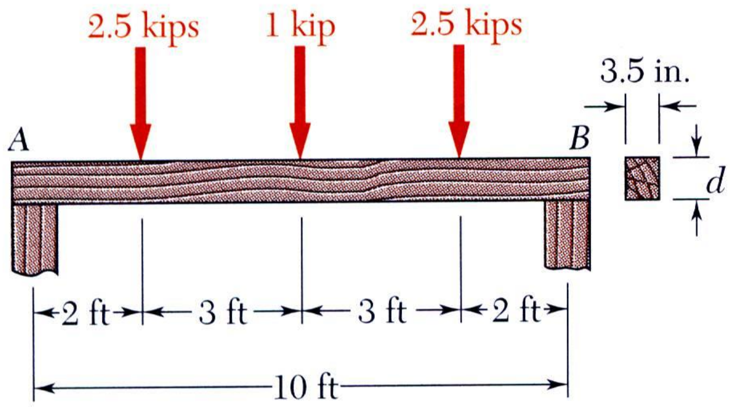


Level	V/I, N/m^4 x 10^9	Q = A (y), m^3	t, m	τ= (V/I)(Q/t), N/m^2
1-1	5.27	0	0.15	0
2-2	5.27	(.03x.15) x (.015+.03)=$2.03\text{x}10^{-4}$	0.15	$7.11\text{x}10^6$
3-3	5.27	(.03x.15) x (.015+0.03)=$2.03\text{x}10^{-4}$	0.03	$35.6\text{x}10^6$
4-4	5.27	(.03x.15) x (.015+.03) + (.03x.03)x0.03/2= $2.2\text{x}10^{-4}$	0.03	$38\text{x}10^6$
4-4	5.27	(.12x.03)(.06)	0.03	$38\text{x}10^6$
5-5	5.27	0		0

Shear stress distribution over the cross section



Sample Problem 2



A timber beam is to support the three concentrated loads shown. Knowing that for the grade of timber used,

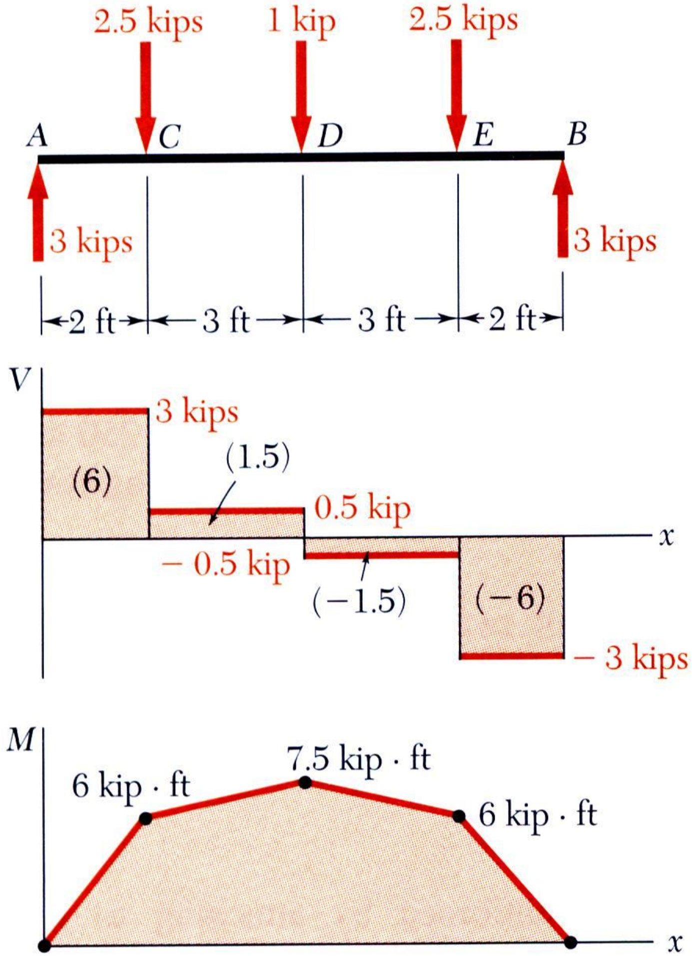
$$\sigma_{all} = 1800\,\text{psi} \qquad \tau_{all} = 120\,\text{psi}$$

determine the minimum required depth *d* of the beam.

SOLUTION:

- Develop shear and bending moment diagrams. Identify the maximums.
- Determine the beam depth based on allowable normal stress.
- Determine the beam depth based on allowable shear stress.
- Required beam depth is equal to the larger of the two depths found.

Sample Problem 6.2



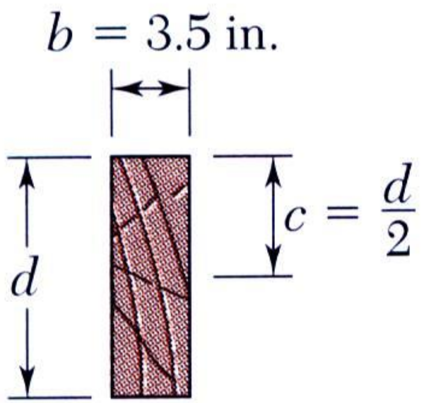
SOLUTION:

Develop shear and bending moment diagrams. Identify the maximums.

$$V_{max} = 3\,\text{kips}$$

$$M_{max} = 7.5\,\text{kip}\cdot\text{ft} = 90\,\text{kip}\cdot\text{in}$$

Sample Problem 6.2



$$I = \frac{1}{12} b d^3$$

$$S = \frac{I}{c} = \frac{1}{6} b d^2$$

$$= \frac{1}{6}(3.5\,\text{in.})d^2$$

$$= (0.5833\,\text{in.})d^2$$

- Determine the beam depth based on allowable normal stress.

$$\sigma_{all} = \frac{M_{max}}{S}$$

$$1800\,\text{psi} = \frac{90 \times 10^3\,\text{lb} \cdot \text{in.}}{(0.5833\,\text{in.})d^2}$$

$$d = 9.26\,\text{in.}$$

- Determine the beam depth based on allowable shear stress.

$$\tau_{all} = \frac{3}{2}\frac{V_{max}}{A}$$

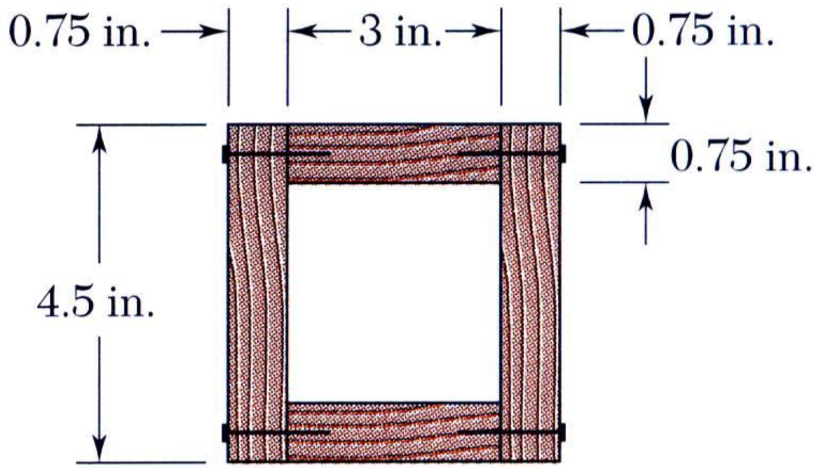
$$120\,\text{psi} = \frac{3}{2}\frac{3000\,\text{lb}}{(3.5\,\text{in.})d}$$

$$d = 10.71\,\text{in.}$$

- Required beam depth is equal to the larger of the two.

$$\boxed{d = 10.71\,\text{in.}}$$

Example 3

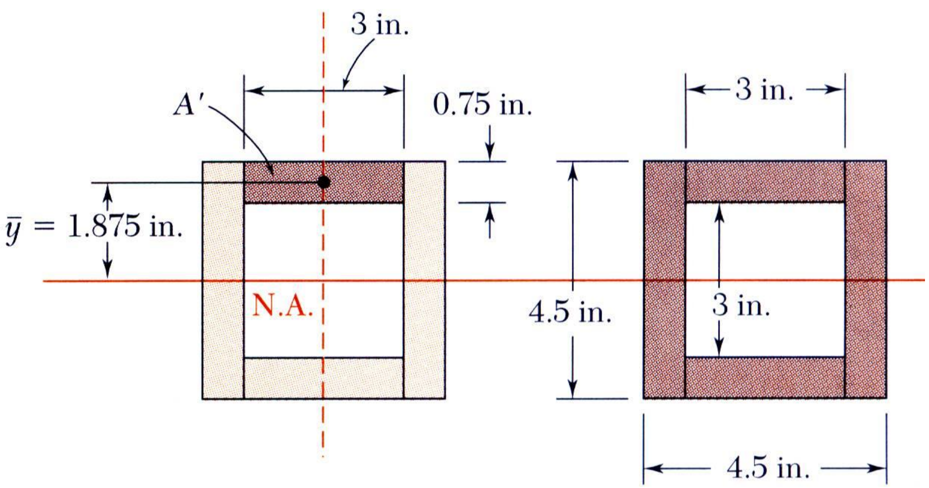


A square box beam is constructed from four planks as shown. Knowing that the spacing between nails is 1.5 in. and the beam is subjected to a vertical shear of magnitude $V = 600$ lb, determine the shearing force in each nail.

SOLUTION:

- Determine the shear force per unit length along each edge of the upper plank.
- Based on the spacing between nails, determine the shear force in each nail.

Example 6.04



For the upper plank,

$$Q = A'y = (0.75\,\text{in.})(3\,\text{in.})(1.875\,\text{in.})$$
$$= 4.22\,\text{in}^3$$

For the overall beam cross-section,

$$I = \tfrac{1}{12}(4.5\,\text{in})^3 - \tfrac{1}{12}(3\,\text{in})^3$$
$$= 27.42\,\text{in}^4$$

SOLUTION:

- Determine the shear force per unit length along each edge of the upper plank.

$$q = \frac{VQ}{I} = \frac{(600\,\text{lb})(4.22\,\text{in}^3)}{27.42\,\text{in}^4} = 92.3\,\frac{\text{lb}}{\text{in}}$$

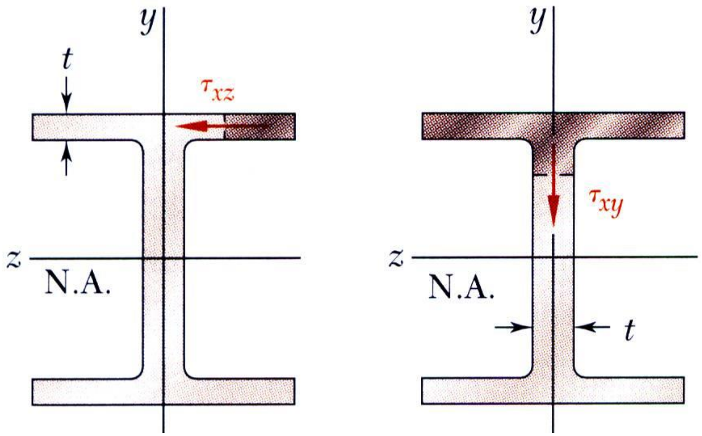
$$f = \frac{q}{2} = 46.15\,\frac{\text{lb}}{\text{in}}$$
$$= \text{edge force per unit length}$$

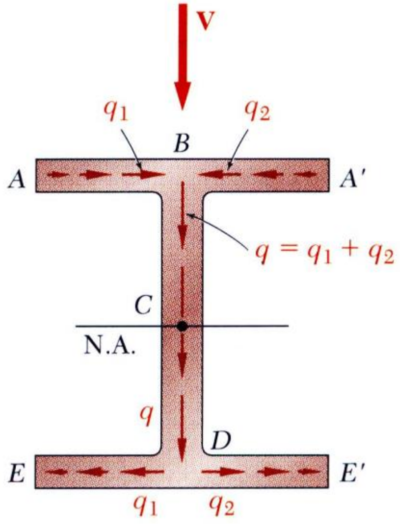
- Based on the spacing between nails, determine the shear force in each nail.

$$F = f\,\ell = \left(46.15\,\frac{\text{lb}}{\text{in}}\right)(1.75\,\text{in})$$

$$\boxed{F = 80.8\,\text{lb}}$$

Shearing Stresses in Thin-Walled Members

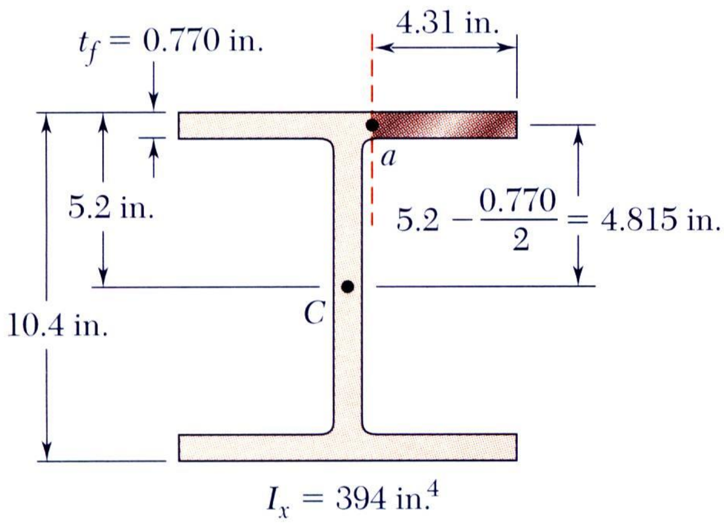




- For a wide-flange beam, the shear flow increases symmetrically from zero at *A* and *A'*, reaches a maximum at *C* and the decreases to zero at *E* and *E'*.

- The continuity of the variation in *q* and the merging of *q* from section branches suggests an analogy to fluid flow.

Sample Problem 3



Knowing that the vertical shear is 50 kips in a W10x68 rolled-steel beam, determine the horizontal shearing stress in the top flange at the point *a*.

SOLUTION:

- For the shaded area,

$$Q = (4.31\,\text{in})(0.770\,\text{in})(4.815\,\text{in})$$

$$= 15.98\,\text{in}^3$$

- The shear stress at *a*,

$$\tau = \frac{VQ}{It} = \frac{(50\,\text{kips})(15.98\,\text{in}^3)}{(394\,\text{in}^4)(0.770\,\text{in})}$$

$$\boxed{\tau = 2.63\,\text{ksi}}$$